PHOTO SCALE

Laboratory Exercise
Air Photo Interpretation & Photogrammetry   Name ____________________
Please Print

Photos: M2330-3, 4, 5 (Course packet)
M1050-79 (From instructor)

"I have neither given nor received unauthorized aid on this assignment."

Signature ______________________________

Objectives

1. To learn procedures for determining photo scale.

2. To understand scale relationships and be able to calculate scale, altitude, elevation, distance, and area given sufficient information.

3. To locate principal points, conjugate principal points, and photo base, and determine the direction of flight.

4. To gain knowledge of the USGS 7.5 minute topographic map series.

Introduction

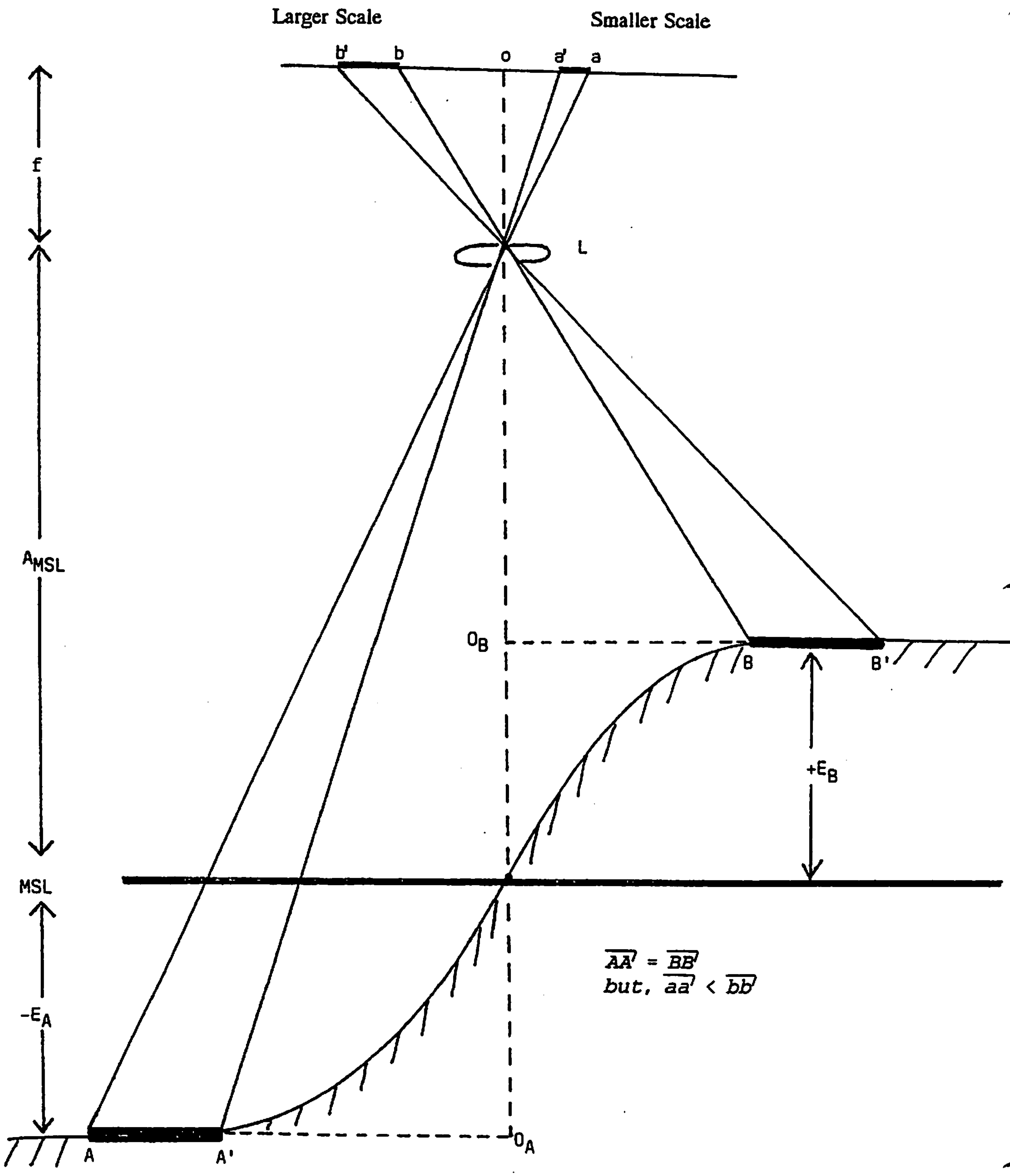
Except under unusual circumstances, aerial photographs do not have a uniform scale. Scale changes continuously with elevation across photographs. Slight tilt in the aerial camera and curvature of the lens can also cause scale variations within a photograph, but changes in ground elevation generally have the greatest impact on scale changes within a photograph.

Consider the example given in Figure 2.1. Lines AA' and BB' are of equal length on the ground and are equidistant from the center of the exposure. However, on the photograph, line bb' will appear longer than aa' because BB' was closer to the camera when the photograph was taken. This is analagous to conventional (terrestrial) photographs in which objects in the background appear smaller than objects in the foreground.

How to Express Scale

1. Representative fraction (ratio factor) - A unitless fraction or ratio of map (or photo) distance to ground distance, always expressed with a "one" in the numerator. For example, a scale of 1:24000 or 1/24000, means one unit on the map equals 24,000 of the <u>same units</u> on the ground. Any units can be used, such as, 1 inch on the map equals 24,000 inches on the ground; or, 1 centimeter on the map equals 24,000 centimeters on the ground. (Note that when expressed as a fraction, the units cancel.)

Figure 2.1. THE EFFECT OF RELIEF ON SCALE



SCALE FORMULA DERIVATION**

AT $\overline{AA'}$: $\dfrac{\overline{aa'}}{\overline{AA'}} = \dfrac{\overline{oL}}{\overline{LO_A}}$

where, $\overline{aa'}$ = *Photo Distance (PD)*

$\overline{AA'}$ = *Ground Distance (GD)*

$\overline{oL}$ = *focal length (f)*

$\overline{LO_A}$ = *Altitude above* $\overline{AA'}$ *or* $A_{MSL} - (-E_A)$

so, $\dfrac{PD}{GD} = \dfrac{f}{A - (-E_A)}$

$$Scale = \frac{f}{A+E_A}$$

AT $\overline{BB'}$: $\dfrac{bb'}{BB'} = \dfrac{oL}{LO_B}$

where, bb' = *Photo Distance (PD)*

BB' = *Ground Distance (GD)*

oL = *focal length (f)*

LO_B = *Altitude above* BB' *or* $A_{MSL} - E_B$

so, $\dfrac{PD}{GD} = \dfrac{f}{A - E_B}$

$$Scale = \frac{f}{A - E_B}$$

** OPTIONAL NOTE (Using "A" as an example):
ΔLoa is similar to ΔLO_AA and $\Delta Loa'$ is similar to $\Delta LO_AA'$
so, $\Delta La'a$ is similar to $\Delta LA'A$

OR

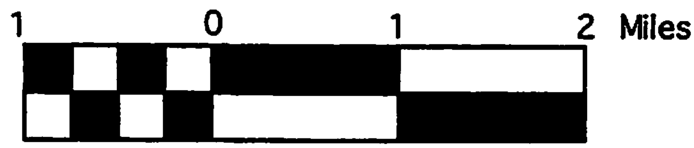
$$\frac{oa}{O_AA} = \frac{oL}{LO_A} \quad \text{and} \quad \frac{oa'}{O_AA'} = \frac{oL}{LO_A} \quad \text{and} \quad \frac{a'a}{oa} = \frac{A'A}{O_AA}$$

$$\text{so,} \quad \frac{a'a}{A'A} = \frac{oL}{LO_A}$$

2. Statement (unit equivalent) - Expressed in terms of two different units of measure, one of which applies to a distance on the map and the other to ground distance. For example, 1" = 2000' means one inch on the map represents 2000 feet on the ground; or, 2.6" = 1 mile means 2.6 inches on the map represents one mile on the ground.

3. Graphic (bar scale) - Small rectangular grid usually showing map distance divisions labelled with the equivalent ground distance. These have one major advantage over other expressions of scale in that the bar scale changes size along with the map if the map is reduced or enlarged.

Example:



Variables

RF = representative fraction
PS = photo scale (S)
PSR = photo scale reciprocal
PD = photo distance
PA = photo area
MS = map scale (S)
MSR = map scale reciprocal
MD = map distance
GD = ground distance
GA = ground area
MSL = mean sea level

AG = above ground
H or A = height or altitude of plane above a reference datum
H_{MSL} = altitude above mean sea level (A_{MSL})
H_{AG} = altitude above the ground (A_{AG})
h or E = height or elevation of an object or point above a reference datum
f = focal length

Scale Formulas

1. $$PS = \frac{PD}{GD} \quad \text{or} \quad MS = \frac{MD}{GD}$$

This is basically the definition of scale: the proportion that a distance on a photo or map represents to a distance on the ground. On a photograph of uneven terrain, this is an average scale between points since actual scale at each point is constantly changing.

2. $$PS = \frac{f}{H_{AG}} \quad \text{or} \quad PS = \frac{f}{H_{MSL} - h}$$

In a vertical aerial photograph, the scale depends on the focal length of the camera and the altitude of the plane <u>above the ground</u>. If the focal length of the camera, altitude of the plane above mean sea level, and elevation of a point on the photo are known, this formula gives the scale at that point. For an entire project or series of photographs, H_{MSL} is often an estimate (the altitude the pilot tried to hold while the pictures were taken) and h is an average elevation of the terrain being photographed; in this instance, this formula represents the <u>target or nominal scale</u> - the desired average scale.

3. $\frac{PS}{MS} = \frac{PD}{MD}$

If the focal length and/or altitude of the plane are not known, an average photo scale can be determined based on reference to a map of known scale.

Sample Calculations

1. Scale Conversions: lines and areas

a. Given an RF of 1:24000, find the ground distance, in feet, for one inch on the photo and find the ground area, in acres, for one square inch on the photo.

1 in. = 24000 in. (one inch PD = 24000 in. GD)

1 in. = 2000 ft.

$(1 \text{ in.})^2 = (2000 \text{ ft.})^2$

1 sq. in. = 4,000,000 sq. ft.

$1 \text{ sq. in.} = \frac{4{,}000{,}000 \text{ sq. ft.}}{43{,}560 \text{ sq. ft./acre}}$

1 sq in. = 91.8 acres (on square inch on the photo= 91.8 acres on the ground)

b. If average photo scale is 1" = .25 miles, find the RF:

1 in. = .25 mi. * 5280 ft./mi.

1 in. = 1320 ft. * 12 in./ft.

1 in. = 15840 in.

RF is 1:15840

2. Scale formula: $PS = \frac{PD}{GD}$

a. Given PD = 2.04 in. and GD = 4080 ft., express scale as an RF. (Note, since an RF always has 1 in the numerator, solve for the denominator):

$$\frac{1}{x} = \frac{2.04 \text{ in.}}{4080 \text{ ft.}}$$

$$\frac{1}{x} = \frac{2.04 \text{ in.}}{4080 \text{ ft.} * 12 \text{ in./ft.}} \quad \text{(units cancel)}$$

$$\frac{1}{x} = \frac{2.04/2.04}{48960/2.04}$$

$$\frac{1}{x} = \frac{1}{24000}$$

RF is 1:24000

b. Given PS = 1:15840 and PD = .87 in, find GD in feet:

$$\frac{1}{15840} = \frac{.87 \text{ in.}}{x}$$

$$x * 1 = .87 \text{ in.} * 15840$$

$$x = \frac{13780.8 \text{ in.}}{12 \text{ in./ft.}}$$

$$GD = 1148.4 \text{ ft.}$$

Note that PD * PSR = GD; PD and GD will be in the same units.

c. Given a rectangular area 2.1 in. by 1.7 in. on the photo, and PS = 1:12000, find the ground area in acres:

$$S^2 = \frac{PA}{GA}$$

Since the right side of the formula is area (distance squared), the left side must also be squared.

$$\left(\frac{1}{12000}\right)^2 = \frac{2.1 \text{ in.} * 1.7 \text{ in.}}{GA}$$

$$\frac{1}{144{,}000{,}000} = \frac{3.57 \text{ sq. in.}}{GA}$$

$$GA * 1 = 3.57 \text{ sq. in.} * 144{,}000{,}000$$

$$GA = 514{,}080{,}000 \text{ sq. in.}$$

$$GA = \frac{514{,}080{,}000 \text{ sq. in.}}{144 \text{ sq. in./sq. ft}}$$

Note conversion:
1 sq. ft. = 144 sq. in.

$$GA = \frac{357{,}000 \text{ sq. ft.}}{43{,}560 \text{ sq. ft./acre}}$$

$$GA = 81.9 \text{ acres}$$

3. Scale formula: $PS = \frac{f}{H_{AG}}$

a. Given a focal length of 153 mm. and a plane altitude of 3000 ft. above level ground, find the average photo scale, expressed as an RF:

$$PS = \frac{153 \text{ mm}}{3000 \text{ ft.}}$$

$$\frac{1}{x} = \frac{153 \text{ mm} \div 25.4 \text{ mm/in.}}{3000 \text{ ft.}}$$

$$\frac{1}{x} = \frac{6 \text{ in.} \div 12 \text{ in./ft.}}{3000 \text{ ft.}}$$

$$\frac{1}{x} = \frac{.5 \text{ ft.}}{3000 \text{ ft.}}$$

$$\frac{1}{x} = \frac{.5/.5}{3000/.5}$$

$$\frac{1}{x} = \frac{1}{6000}$$

PS = 1:6000

b. Given f = 6 in. and an average ground elevation of 350 ft., at what altitude above MSL should the plane be flown in order to achieve an average scale of 1 in. = 250 ft.?

PS is 1 in. = 250 ft. or 1:3000

$H_{AG} = H_{MSL}$ - 350 ft. (Altitude of the plane above the ground equals altitude of plane above mean sea level minus the elevation of the ground)

So, $\frac{1}{3000} = \frac{6 \text{ in.}}{H_{MSL} - 350 \text{ ft.}}$

$$\frac{1}{3000} = \frac{.5 \text{ ft.}}{H_{MSL} - 350 \text{ ft.}}$$

$$1 * (H_{MSL} - 350 \text{ ft.}) = .5 \text{ ft.} * 3000$$

$$H_{MSL} - 350 \text{ ft.} = 1500 \text{ ft.}$$

$$H_{MSL} = 1500 \text{ ft.} + 350 \text{ ft.}$$

$$H_{MSL} = 1850 \text{ ft.}$$

4. Scale formula: $\frac{PS}{MS} = \frac{PD}{MD}$

Given an aerial photograph taken over level terrain and a 1:20,000 scale map of the same area, a series of measurements are made between points that can be found on both the photo and the map. Find the average scale of the photo given the following measurements:

Line	PD	MD
1	1.00"	1.79"
2	2.03"	3.66"
3	1.57"	2.91"
4	1.32"	2.35"

$PS = \frac{1}{x}$ where x = PSR

$$MS = \frac{1}{20{,}000}$$

PD(1) = 1.00"

MD(1) = 1.79"

For line #1:

$$\frac{\frac{1}{x}}{\frac{1}{20,000}} = \frac{1.00\ \text{in.}}{1.79\ \text{in.}}$$

$$\frac{20,000}{x} = \frac{1.00}{1.79}$$

$$x * 1.00 = 1.79 * 20,000$$

$$x = 35,800$$

Perform the same calculation for the remaining lines, then average the PSRs.

Line	PSR
1	35800
2	36059
3	37070
4	35606

$\bar{x}$ = 36134

So the average RF is 1:36134

"Smaller" and "Larger" Scales

"Small scale" and "large scale" are only relative terms, but you should know the difference.

Large scale means objects or features appear large on the photo or map. The RF is a large fraction. The PSR (i.e. the denominator of the RF) is a small number. Small scale means objects or features appear small on the photo or map. The RF is a small fraction. The PSR is a large number.

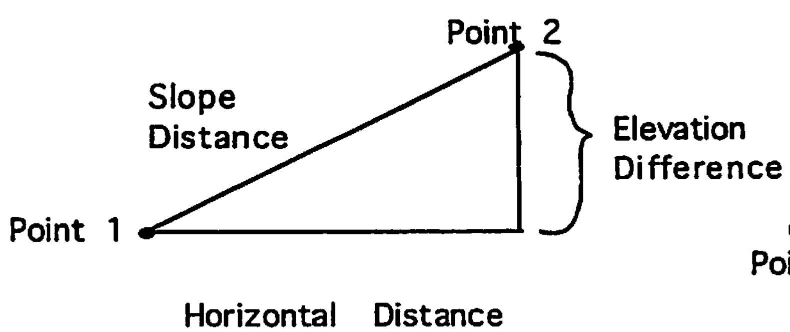
Thus, 1:36000 is a smaller scale than 1:24000 (36000 is a larger number than 24000 and 1/36000 is a smaller fraction than 1/24000.) Small scale photographs cover more ground area in each frame than large scale photographs, but everything looks smaller.

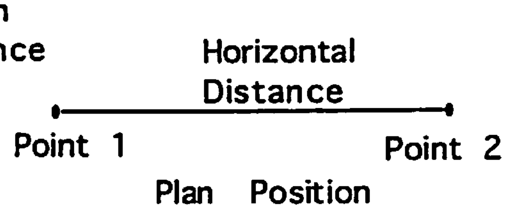
Again, there are no fixed limits for what is called "small" versus "large" scale, but when dealing with aerial photography, the following limits might be applied:

1. Large scale - 1:12000 and larger
2. Medium scale - 1:12000 to 1:36000
3. Small scale - 1:36000 and smaller

USGS Map Series

The U.S. Geological Survey (USGS) is the major civilian mapping agency of the United States. The USGS produces many different map series in both map and digital (computer compatible) forms. One of the most commonly used series is the 7 1/2 minute (7.5') topographic map series. Each map in this series is called a quadrangle and covers an area which is 7.5' of latitude or longitude on a side. Each quadrangle is named (e.g. Raleigh West, N.C.) and has a scale of 1:24000. In addition to latitude and longitude, tick marks for Universal Transverse Mercator (UTM in meters) and State Plane (SP in feet) coordinates are also provided. These maps are planimetric; that is, objects or features shown on the maps appear in their true, plan position. Distances measured from the maps are horizontal distances, not slope distances.





Contour lines connecting points of equal elevation and features such as roads, building, marshes, and drainages are depicted on the maps. USGS 7.5' topographic quadrangles meet National Map Accuracy Standard requirements which specify the amount of allowable horizontal and vertical error.

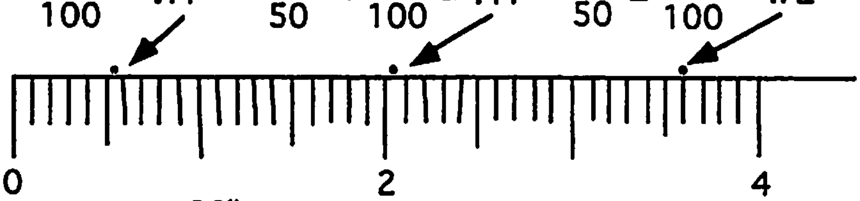
Exercises

UNITS: 1 ft. = 12 in., 1 in. = 2.54 cm, 1 acre = 43,560 square ft.
1 mile = 5280 ft. 1 meter = 3.28 ft.

Always include units in your answers, when appropriate.

NOTE: Unless otherwise specified, photo measurements should always be made to the nearest hundredth of an inch (.01"). Use the scale marked 50 on the engineer's ruler. Each mark represents 1/50 of an inch (.02"). Between marks is .01". Longer lines are 10/100" or a tenth of an inch.

Examples: $\frac{5.5}{50} = \frac{11}{100} = .11''$ $\frac{20.5}{50} = \frac{41}{100} = .41''$ $\frac{36}{50} = \frac{72}{100} = .72''$



$$2 = \frac{2''}{5} = \frac{20''}{50} = \frac{40''}{100} = .4''$$

$$4 = \frac{4''}{5} = \frac{40''}{50} = \frac{80''}{100} = .8''$$

(6 pts) 1. When aerial photographs are taken, focal length and average ground elevation are taken into consideration, and the plane is flown at an altitude (generally referenced to MSL) designed to achieve a particular average project scale. Often, the flying altitude and/or the target, or nominal, scale appear in the photo annotation. It should be understood that the nominal scale applies only in a very general way to the entire project and probably does not apply to any one photograph or feature. To improve estimates of distances or areas measured directly on a photograph, scale should be calculated at a particular point of interest or an average scale should be determined for individual photographs. Note, however, that ground measurements made directly from a photograph will still have some error.

Using the Raleigh West and Linville Falls 7.5' quads as reference, follow the procedure described in class and compute the average scale for photos 5, M2330 (in your course packet) and 79, M1050 (from your instructor). Use ratios of at least four line measurements on each photo and the map. On the photos, label the end points of the lines and record these lines below. Lines on the photos should be one to three inches in length, perpendicular to radial lines from the centers of the photos, and should be distributed around the photos. Lines used to scale photo 5 should be at relatively equal elevations.

Line	Map Distance	Photo 5 Dist.	Scale
AB			
CD			
EF			
GH			

Average ____________________

	Map Distance	Photo 79 Dist.	Scale
ab			
cd			
ef			
gh			

Average ______________________

(1 pt.) 2. Which photograph, 5 (M-2330) or 79 (M-1050), has the larger scale?

(1 pt..) 3. Convert your average scale on photo 79 to inches/mile (i.e. how many inches on the photo are equivalent to one mile on the ground?)

(2 pts.) 4. a. If the camera that photo 79 was taken with has a focal length of 8.25 inches, at what altitude, in feet, above the terrain was this photo taken?

b. At approximately what altitude MSL was the plane flying? (HINT: Use the topographic map as a reference.)

Average h __________ H_{MSL} __________

(1 pt.) 5. If the ground distance between two points is 50 meters, what will the approximate distance be between the same two points on photo 5 (M-2330), in inches?

(2 pts.) 6. What should be the dimensions, on your photo (in inches), of a square piece of land 160 acres in size situated close to the center of photo 5 M-2330

(4 pts.) 7. Photographs are to be taken for preparing a highway design map in Eastern North Carolina. The lowest elevation in the area to be photographed, Point A, is 1,110 ft., and the highest elevation Point B, is 2,000 ft. The minimum (smallest) acceptable photographic scale is to be 1:6000 and the camera to be used contains a lens with a focal length of 152.4 mm.

a. Draw a diagram of these conditions. Label all variables including: Points A and B, H_{MSL}, H_{AG}, f, h_a, h_b (where h_a and h_b are the elevations of points A and B, respectively), and the reference datum (MSL). You do not need to use numbers in the diagram, just variable names (or abbreviations).

b. What must be the flying height above mean sea level in order to meet the minimum scale requirement?

c. What will be the maximum (largest) scale?

Principal Points and Conjugate Principal Points

The principal point (PP) is the geometric center of an aerial photograph and is found by drawing intersecting lines from the fiducial marks on the sides of the photograph. On a truly vertical photograph, the PP and nadir are the same. Unless otherwise specified, in this course, we assume all photographs are truly vertical. The PP (nadir) also shows where a vertical photograph was taken, since the camera and plane are directly above the center of the picture when the exposure is made.

With photographs which overlap more than 50% (60% is a standard) along a flight line, each PP also appears on the two adjacent photographs. The location of a PP on an adjacent photograph is a conjugate principal point (CPP). In a flight line, every photograph except the end photos will have one PP and two CPPs.

The line which connects the PP to the CPP is called the photo base (PD) or flight base (GD). This line also shows the path the plane flew between exposure stations: thus, it is also the flight line. Since individual photographs are numbered sequentially, proper labelling of the PP and CPPs will show you on the photo, which direction the plane was flying.

When inspecting photographs for the first time, read the annotation on the photograph. At a minimum, this should tell you when the photograph was taken (date) and the frame number. (If the photographs were taken with 80% overlap, every other photograph will provide 60% overlap - enough for stereo viewing; but, in most cases, 60% overlap is flown and frame numbers must be consecutive for stereo viewing.)

Line up your ruler between fiducial marks on opposite sides of the photographs and mark the PP with a cross mark in the center of the photo. Do <u>not</u> draw the lines completely across the photographs - just in the center. All other points must be marked with a barely discernable dot with a circle around it.

Locate and mark the CPPs. Sometimes this can be done by inspecting the detail in both photographs and finding identical points on the ground. However, it is best to transfer the PP while viewing the photographs in stereo and this is the approach you should use. Note that placement should be checked (non-stereo). Label all points based on the photograph from which they came. Do not draw lines <u>through</u> the points.

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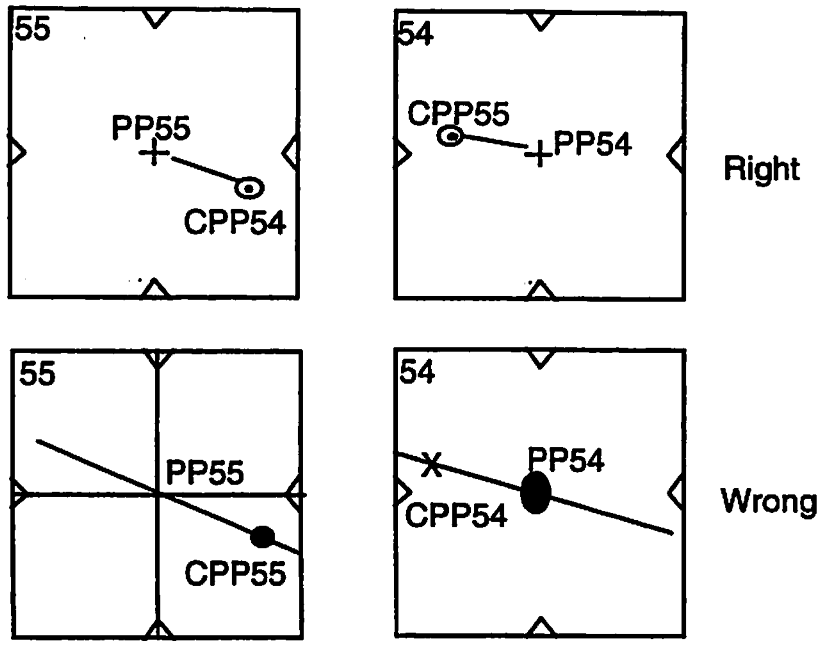
Example:



For precise location of points, photogrammetrists normally prick the photograph with a needle. Because this is a destructive method, we do not require that you pin prick points. However, after practicing point transfer with a grease pencil, you can prick points on photographs you have purchased.

(3 pts.) 8. a. Locate the principal points and the conjugate principal points on the photos (M-2330 4 and 5) and measure the photo bases (the distance between pp and cpp) on photos 4 and 5.
Label points carefully on the photos.

Photo base (4) _____ inches Photo base (5) _____ inches

b. What is the average length of these photo bases on the ground, in feet? (You can assume photo 4 has the same average scale as photo 5.)

Turn in photos 4 and 5 with the lab.

Vocabulary

Scale - The proportion that a distance on a photo or map represents to a distance on the ground.

Average Scale - On a photograph, the scale between two points, the average for an entire photograph, or an average for the entire project.

Nominal Scale (target scale) - The desired average scale for a photograph or project.

Point Scale - Scale at a point on the ground at a known elevation.

Principal Point - Geometric center of the photograph at the intersection of the x and y axes of the photo.

X-Axis - Line which connects fiducial marks on opposite sides of the photograph and which is most nearly parallel to the flight line.

Y-Axis - Line which connects fiducial marks on opposite sides of the photograph and is perpendicular to the x-axis.

Conjugate Principal Point - The location of a principal point on an adjacent, overlapping photograph.

Nadir - The point vertically beneath the center of the camera lens at the time of exposure.

Fiducial Marks - Geometric figures (dots, crosses, half arrows, etc.) optically projected onto the sides and/or corners of a photograph and used as reference marks to define the x and y axes and the PP.

Overlap - The area covered by two consecutive or side-by-side photographs. The area of common coverage is usually expressed as a percentage of each photo.

Photo Base (air base) - The length (PD or GD) of an imaginary line connecting points at which successive photos in a flight strip were taken.

skk[Scale.Exam]